Physics 105

2025-2024

Tabarak Al-Rahmman

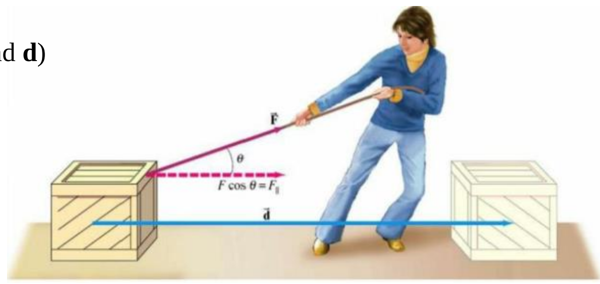
ARKAN ACADEMY

Chapter -6-
(Worked Energy)

❖ Section (6.1): Work done by a constant force

- To move the box a displacement $\vec{d}$ to the right as shown in the figure, work must be done.
  - ➢ Work done: Magnitude of the displacement times the component of the force parallel to the displacement, this is a scaler quantity

    $\mathbf{W} = (\mathbf{F}\cos\theta)\,\mathbf{d} = F\,d\,\cos\theta$ (Where **θ** is between **F** and **d**)

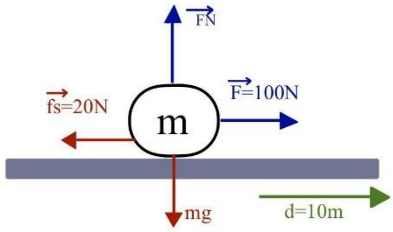


- So, when the work equals zero:
  - ➢ $\theta = 90^{\circ}$
  - ➢ $d = 0$
- The Unit of work : [W] = N.m = Joule

✓ ***Example***: In the figure , the box moves a distance of 10 m to the right .

I. Find the work done by each force

II. Find the net (total) work done on the box



✓ ***Solution:***

I.

$W_{F_N} = F_N\, d\cos(90^{\circ}) = 0$

$W_{mg} = mg\, d\cos(90^{\circ}) = 0$

$W_F = F\, d\cos(0^{\circ}) = F\, d = (100)*(10) = 1000\, J$

$W_{f_k} = f_k\, d\, \cos(180^{\circ}) = (20)*\,(10)*\,(-1) = -200\, J$

II.

$W_{net} = W_{F_N} + W_{mg} + W_F + W_{f_k}$

$= 0 + 0 + 1000 + (-200) = 800\, J$

★ ***Question1***: What does the work *greater* than zero mean?

☑ ***Solution:***

It means that the force moves and accelerating the object .

★ ***Question2***: What does the work *less* than zero mean?

☑ ***Solution:***

It means that the friction force *impedes* the motion of the box and decelerating the object , as we know the work is a scaler quantity so the negative sign *doesn't* mean direction.

★ ***Question3***: What does the work *equals* zero mean?

☑ ***Solution:***

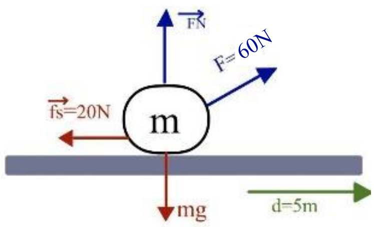
It means that the object moves in a constant speed.

- When we have a ***positive*** work: transfers energy **to** the object.
- When we have a ***negative*** work: transfers energy **from** the object.

★ ***Question4***: Find the net work done by the box

(when F = 60N and the θ of F is 30˚)

☑ ***Solution:*** ($W_{net} = 159.80\, J$)



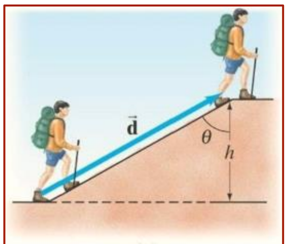
✓ ***Example :***

I. Determine the work in a hiker must do on a 15.0 Kg backpack to carry it up a hill of height h= 10m.

II. Determine the work done by gravity on the backpack.

III. Determine the net work done on the backpack.

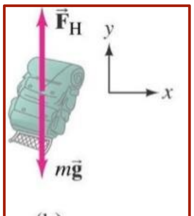
(*Assume the motion is* ***smooth*** *and at* ***a constant velocity***)



(a)

✓ ***Solution***

I. First of all draw a free body diagram then choose a coordinate system and apply the newton's second law



(b)

$\sum F_y = ma_y$

$F_H - mg = 0$

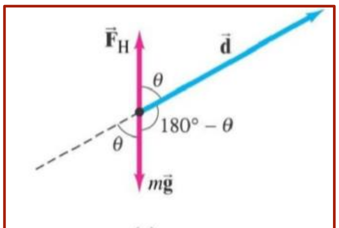
$F_H = mg$ (which the F_H is a force the hiker must exert upwards to support the backpack)

$F_H = 15 * 9.8$

$F_H = 147\ N$

$W_{F_H} = (F_H) * (d)\cos\theta = 147 * h = 147 * 10$

$= 1470\ J$



(c)

II. $W_{mg} = F_H(d)\cos(180^\circ - \theta) = F\_H(-d\cos\theta) = 147 * -10 = -1470\ J$

III. $W_{net} = W_{F_H} + W_{mg} = 0$ (Because the object is moving in a constant velocity)

★ ***Question 5*:** Does the Earth do work on the moon?

☑ ***Solution:***

No, the work done by *Earth gravity* is zero because the angle between the displacement and the gravitational force is **90°**, So cos ($90^\circ = 0$)

❖ Section (6.3): Kinetic Energy and the work – Energy principle –

- Kinetic Energy:
  - ➢ It is a form of energy associated with the motion of a moving object, this ***never be negative*** and it is a scalar quantity.

$$KE = \frac{1}{2}\, m\, v^2$$

- The Unit of kinetic energy (KE) is Joule
- When $KE = 0$ then the object is at rest
- **Work – kinetic energy theorem:** The total (net) work done on the object **equals** the change in its kinetic energy.

$$W_{net} = \Delta KE$$

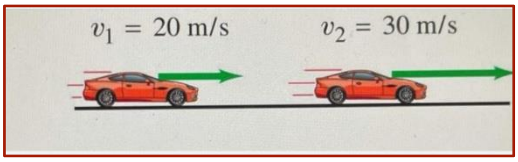
$$W_{net} = KE_f - KE_i$$

$$W_{net} = \frac{1}{2}\, m\, (V_f^2 - V_i^2)$$

- When $W_{net} = 0 \rightarrow \Delta KE = 0 \rightarrow KE_f = KE_i \rightarrow V_f = V_i$
  - ➢ This means that the speed of the object doesn't change
- When $W_{net} > 0 \rightarrow KE_f > KE_i \rightarrow V_f > V_i$
  - ➢ This means that the object is accelerating.
- When $W_{net} < 0 \rightarrow KE_f < KE_i \rightarrow V_f < V_i$

- This means that the object is decelerating.

✓ ***Example:*** How much work is required *to accelerate* a 1000 kg car from 20m/s to 30 m/s?



✓ ***Solution:***

We use the work energy theorem $W_{net} = \Delta KE$

$W_{net} = KE_f - KE_i$

$W_{net} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$

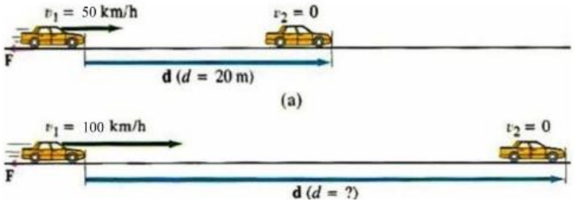
$W_{net} = \frac{1}{2} m (v_f^2 - v_i^2) = \frac{1}{2} \times 1000 (900 - 400) = 25 * 10^4 J$

✓ ***Example:*** A car traveling 50 Km/h can brake to a stop in a distance d of 20 m. If the car is going *twice as fast*, 100 Km/h,What is its stopping distance? (Assume the maximum braking force is approximately independent of speed).

✓ ***Solution:***

$W_{net} = Fd\, cos\theta = Fd\, cos180^\circ = -Fd$

So the work – energy theorem



$W_{net} = \Delta KE$

$W_{net} = KE_f - KE_i$

$- Fd = KE_f - KE_i$

$- Fd = \frac{1}{2} m ({v_f}^2 - v_i^2)$

$- Fd = \frac{1}{2} m (0 - v_i^2)$

$Fd = \frac{1}{2} m (v_i^2)$

$d = \frac{m}{2F} v_i^2$ (the mass and force are constants)

$d \ \alpha \ v_i^2$

So the $v_i' = 2 v_i$

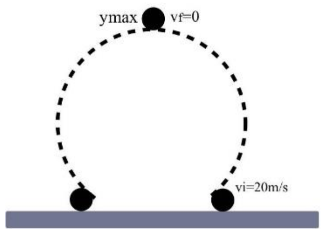
And the $d' = \frac{m}{2F} (v_i')^2 = \frac{m}{2F} (2 v_i)^2$

$d' = 4 \left(\frac{m}{2F} v_i^2 \right)$

$d' = 4 d = 4 \times 20$

$\mathbf{d'} = 80 m$

✓ ***Example:*** An object of mass is projected *vertically upwards* from the Earths' surface with an initial speed of 20 m/s. Find its maximum height.



✓ ***Solution:***

Free fall body, The only force acting on the object is weight downwards and the displacement $\vec{d}$ is upwards so the angle $\theta = 180^\circ$

$W_{net} = \Delta KE$

$\cancel{m} g (y_{max}) cos180^\circ = \frac{1}{2} \cancel{m} (v_f^2 - v_i^2)$

$- g\, y_{max} = \frac{1}{2} (0 - 20^2)$

$g\, y_{max} = 200 \longrightarrow y_{max} = 20.408\, m$

❖ Section (6.4): Potential energy

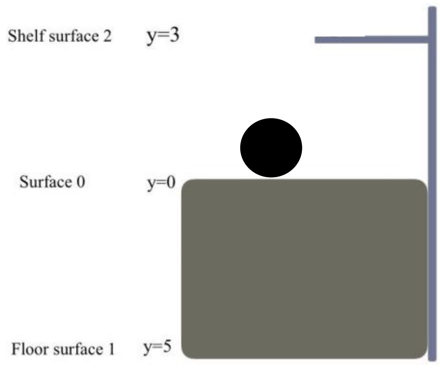
- Gravitational potential energy: Is a form of energy associated with the height of object relative to the surface of the Earth and it is a scalar quantity.

$$U = mgh$$

$$PE_g = mg\,y$$

- The Unit of Potential energy(PE) is Joule.

✓ ***Example:*** Gravitational potential energy is defined with respect to a surface (When we remove the table) like :



I. The potential energy of the book relative to shelf surface 2.

$$U_2 = -\ m\ g\ h_2$$

(Negative U_2 means work must be done to raise book to shelf)

II. The potential energy of the book relative to floor surface 1.

$$U_1 = \ m\ g\ h_1$$

(Positive value means if you release book it falls towards floor)

I. The potential energy of the book with respect to surface 0.

The book is on the surface so $h = 0$, $U_0 = 0$

- ***Unlike*** kinetic energy, the potential energy could be ***positive***, ***negative*** or ***zero***.
  - ➢ Work – energy theorem can be written as:

$$W_F = \Delta U$$

$$W_F = U_f - U_i = m\,g\,(y_f - y_i)$$

✓ ***Example:*** A ball is held 3m above *the edge* well and then dropped into it. The well has a *depth* of 5m , choosing the *top edge* of the well as the $y = 0$ point of your coordinates system , what is U of the ball..

I. Before it is released?
II. When it reaches the bottom of the well?
III. What is ΔU from release to reaching the bottom of the well?
IV. What is ΔKE from the release position to the bottom of the well position?

✓ ***Solution:***

I. $U_1 = mg\,y_1 = (3\,mg)$ J

II. $U_2 = mg\,y_2 = (-5\,mg)$ J

III. $\Delta U = U_2 - U_1 = -5\,mg - +3\,mg = (-8\,mg)$ J

IV. $\Delta\,KE = \frac{m}{2}\,(v_f^2 - v_i^2)$

To find v_f we use equations of motion

$$v_f^2 = v_i^2 + 2\,g\,\Delta\,y$$

$$v_f^2 = 0 + 2\,(-9.8)\,(-5-3) = 2\,(-9.8) * (-8)$$

$$\sqrt{V_f^2} = \sqrt{156.8} \longrightarrow v_f = 12.52\,\frac{m}{s}$$

$$\Delta\,KE = \frac{m}{2}\,(16\,g - 0)$$

$$\Delta\,KE = (8\,mg)\,J$$

When we calculate the W_{ext} from the previous example we get:

$W_{ext} = F_{ext}\ d\ \cos\theta$ (and $\theta = 0^{\circ}$)

$W_{ext} = F_{ext}\ d$

$W_{ext} = mg\,d = mg\ (y_2 - y_1)$

$W_{ext} = \Delta U$

And calculate the W_G

$W_G = -\,m\,g\,d$

$W_G = -mg\,(y_2 - y_1)$

$W_G = -\,\Delta\ U$

So the W_{net} equals

$W_{net} = W_{ext} + W_G$

$= \Delta U + -\Delta U$

$= 0$ → (Which means that the object moves at a constant speed)

❖ Section (6.5): Conservative and Nonconservative Force

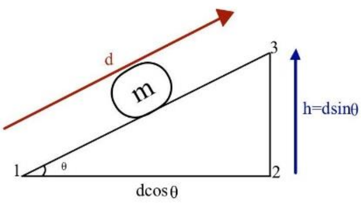
- Conservative Forces: A forces which their work done ***doesn't depend*** on the path taken , but only on the *initial* and *final positions* and their work around a ***closed path*** is zero.
- **Examples** of Conservative forces:
  - **I.** Gravitational force.
  - **II.** Spring force.
  - **III.** Electric force.
- Nonconservative Forces: A forces which their work done ***depends*** on the path , and their work around a ***closed path*** isn't equal zero
- **Examples** of Nonconservative forces:
  - **I.** friction
  - **II.** Push or pull by a person
  - **III.** Tension in cord

✓ ***Example:*** Find the work done by the force of gravity when the box of mass m moves from point 1 to 3 through two different paths.

I. 1 to 3 directly.

II. 1→2→3

III. 1→2→3→1 (closed path)



✓ ***Solution:***

I. $W_{mg}(1 \rightarrow 3) = mg\sin\theta\ d\ \cos\theta$ (where the angle equals 180°)

$W_{mg}(1 \rightarrow 3) = -\,mg\ d\ \sin\theta$ ($h = d\ sin\theta$)

$W_{mg}(1 \rightarrow 3) = -\,mg\ h$ → $W_{mg} = -\,\Delta\ U$

II. $W_{mg}(1 \rightarrow 2 \rightarrow 3) = W_{mg}\ (A \rightarrow B) + W_{mg}\ (B \rightarrow C)$

$= mg\ d\ cos90^{\circ} + mg\ h\,cos180^{\circ}$

$= 0 + -mg\ h$

$= -mg\ h = -\,\Delta\ U$

The work of gravity doesn't depend on the path.

III. Closed path w_{mg} (1 → 2 → 3 → 1)

$$w_{mg}(1 \rightarrow 2 \rightarrow 3 \rightarrow 1) = w_{mg}(1 \rightarrow 2 \rightarrow 3) + w_{mg}(3 \rightarrow 1)$$

$$= -mg\,h + mg\,sin\theta\;d\;cos\theta \quad \text{(where } \theta = 0^\circ\text{)}$$

$$= -mg\,h + mg\,h \quad (h = d\,sin\theta)$$

$$= 0$$

- The work of gravity around a closed path = 0
- Work – Energy extended

$$\boldsymbol{W_{net}} = \Delta\,\boldsymbol{KE}$$

- We know $W_{net} = W_c + W_{NC}$
- (W_c work done by conservative force on object)
- (W_{Nc} work done by Nonconservative force on object)

So

$$\boldsymbol{W_c} + \boldsymbol{W_{NC}} = \Delta\,\boldsymbol{KE} \quad \text{(We know } \boldsymbol{W_c} = -\Delta\,\boldsymbol{U}\text{)}$$

$$-\Delta\,\boldsymbol{U} + \boldsymbol{W_{NC}} = \Delta\,\boldsymbol{KE} \qquad \boldsymbol{or} \qquad \boldsymbol{W_{NC}} = \Delta\,\boldsymbol{KE} + \Delta\,\boldsymbol{U}$$

✓ The work done by nonconservative forces acting on an object is equal to the total change in kinetic and potential energies

❖ Section (6.6): Mechanical energy and its conservation

- Total Mechanical energy (E): The sum of the kinetic and potential energies at any moment is a scalar quantity.

$$\boldsymbol{E} = \boldsymbol{KE} + \boldsymbol{PE}$$

✓ E : Total mechanical energy
✓ KE : Kinetic energy
✓ PE : Potential energy

- When no nonconservative force do work, then the $\boldsymbol{W_{NC}} = \boldsymbol{0}$

$W_{net} = 0$

$W_{net} = \Delta\,KE + \Delta\,U = (KE_f - KE_i) + (U_f - U_i)$

$(KE_f + U_f) - (KE_i + U_i) = 0$

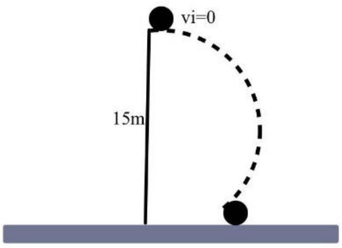
$E_f - E_i = 0$ (Which means $E_f = E_i = constant$)

$\Delta\,\boldsymbol{E} = \boldsymbol{0}$

- There is ***no loss*** of total mechanical energy E , that means total mechanical energy *is conserved*
  - All of the above laws can be applied only when no nonconservative forces are present.

❖ Section (6.7): **Problem solving using conservation of mechanical energy**

✓ ***Example***: An object of mass m is dropped from a height of 15 m above the Earths' surface, (Ignoring air resistance) Find its speed just before hitting the ground.



✓ ***Solution:***

The gravitational force only acts on object, so it is a conservative force. Therefore, we can use [total mechanical energy is conserved]

$$\Delta \boldsymbol{KE} + \Delta \boldsymbol{U} = \mathbf{0}$$

$\Delta U =$ If object rises $\longrightarrow$ $\Delta U = mg\,h$

$=$ If object falls $\longrightarrow$ $\Delta U = -mg\,h$

$=$ If object remains on the same level $\longrightarrow$ $\Delta U = 0$

So, If we go back to

$\Delta \boldsymbol{KE} + \Delta \boldsymbol{U} = \mathbf{0}$

$\frac{1}{2}\cancel{m}\left(v_f^2 - v_i^2\right) + -\cancel{m}g\,h = 0$

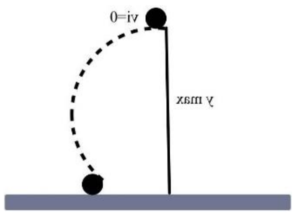
$\frac{1}{2}\left(v_f^2 - 0^2\right) + -g\,(15) = 0$

$\frac{1}{2}\left(v_f^2\right) + -15g = 0$

${v_f}^2 = 30\,g$

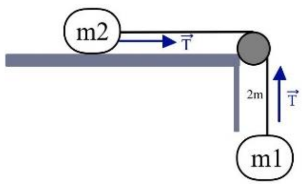
$v_f = \sqrt{30\,g}$ $\longrightarrow$ $v_f = 17.14$ m/s

★ ***Question 6:*** A ball is projected vertically upwards from ground level with an initial speed of 20 m/s. (Ignoring air resistance) Find its maximum height.



☑ ***Solution:*** ($y_{max} = 20$ m)

✓ ***Example:*** Find the speed of the ball 1 when it has fallen a vertical distance of 2 m (Assume system started from the rest and all surfaces are smooth).



✓ ***Solution:***

Only force acting in the system is gravitational force, why? Because

- the tension force in *vertical* has a work done in *negative*:

  $W_{T_{vertical}} = T\,d\cos 180^\circ = -T\,d$

- While in *horizontal* has a work done in *positive*:

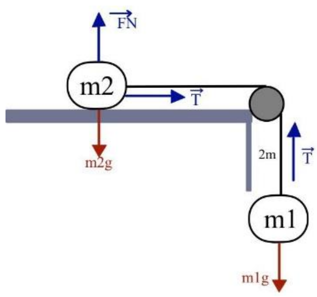
  $W_{T_{horizontal}} = T\,d\cos 0^\circ = T\,d$

- So, the total work done by the tension is

  $W_{total} = W_{T_{vertical}} + W_{T_{horizontal}}$

  $= -T\,d + T\,d$

  $= \mathbf{0}$



So, the gravitational force only acting in the system so the *total mechanical energy* is *conserved:*

$$\Delta\, \boldsymbol{KE} + \Delta\, \boldsymbol{U} = \mathbf{0}$$

and for two balls $(\Delta\, KE_1 + \Delta\, U_1) + (\Delta\, KE_2 + \Delta\, U_2) = 0$

$$\frac{1}{2}\, m_1 \left(v_{1_f}^2 - v_{1_i}^2\right) + -m_1\, g\, h + \frac{1}{2}\, m_2 \left(v_{2_f}^2 - v_{2_i}^2\right) + 0 = 0$$

- Note that $\Delta\, U_2 = 0$ because there is no change in height

$$\frac{1}{2}\, m_1 \left(v_{1_f}^2 - 0\right) + -2m_1 g + \frac{1}{2}\, m_2 \left(v_{2_f}^2 - 0\right)$$

The balls 1 and 2 are connected by a string.

So, $v_{1_f} = v_{2_f} = v_f$

$$\frac{1}{2}\, m_1 {v_f}^2 + -2\, m_1 g + \frac{1}{2}\, m_2\, v_f^2 = 0$$

$$\frac{1}{2}\, (m_1 + m_2)\, {v_f}^2 - 2\, m_1 g = 0$$

$$\frac{1}{2}\, (m_1 + m_2)\, {v_f}^2 = 2\, m_1 g$$

$${v_f}^2 = \frac{4m_1 g}{m_1 + m_2}$$

$$v_f = \sqrt{\frac{4m_1 g}{m_1 + m_2}}$$

❖ Section (6.8): Other forms of energy transformations the law of conservation of energy

- In this chapter, we learned about the separation of energy from food and potential energy, and also mentioned some other ***types of energy***, such as *electrical*, *sensitive*, and *chemical energy* stored in food and fuel. These other types of energy, with the ***exception of kinetic energy***, are ***considered potential energy***. For example, energy is stored in food or fuel, such as gasoline. It is stored as potential energy on In contrast to thermal energy, it is the movement of molecules with unique formations, so we have many energy conversions from potential to kinetic and vice versa, and because the relationship between work exists: work is accomplished when energy is *transferred* from ***one body to another***.
- The law of conservation of energy is one of the **most important** principles in physics.
- (The total energy is neither increased nor decreased in any process) But the validity of this law is encompassing *all forms of energy* **including** those associated with *nonconservative force*

❖ Section (6.9): Energy conservation with dissipative force: solving problems

- Dissipative force: It is the force that ***resists*** and ***dissipates*** energy, like *friction* and *tension.*
- The work done by such nonconservative force W_{NC} is taken into account using

$$\Delta\, KE + \Delta\, U = W_{NC}$$

$$\left(KE_f - KE_i\right) + \left(U_f - U_i\right) = W_{NC}$$

$$\left(KE_f + U_f\right) - \left(KE_i + U_i\right) = W_{NC}$$

$$E_f - E_i = W_{NC}$$

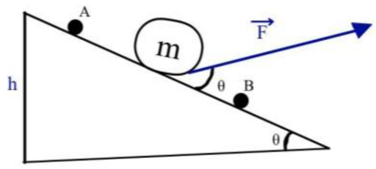
$$\Delta\, \boldsymbol{E} = \boldsymbol{W}_{\boldsymbol{NC}}$$

  - $\Delta\, E$: Change in total mechanical energy
  - W_{NC} : Work done by nonconservative force

✓ ***Example:*** The ball is sliding down in the incline plane from A to B. $\vec{F}$ is applied between A and B and $KE_A = 10$ J , $KE_B = 20$J , $\theta = 37^\circ$, $\overrightarrow{d_{A\to B}} = 5$m , $\mathbf{F} = 10$ N and m = 4 kg.

- How much work is done on the ball by f_K between A and B.

✓ ***Solution:***



In this example we use this equation $\boldsymbol{W_{net}} = \Delta\, \boldsymbol{KE} + \Delta\, \boldsymbol{U}$

And we know that $W_{net} = W_{NC} + W_F$

$W_{NC} = \Delta\, KE + \Delta\, U - W_F$

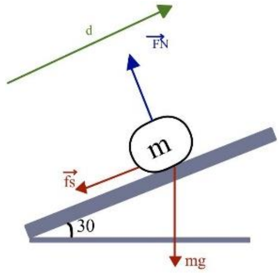
$= (KE_B - KE_A) + (mg\, h) - (F\, d \cos\theta\, \cos\emptyset)$ (Where $\emptyset = 180^\circ$)

$= (20 - 10) + (mg\, (-d \sin\theta\,) + (F\, d\cos\theta\,)$

$= (10) + (- 117.955) + (39.93)$

$= -68.023$ J

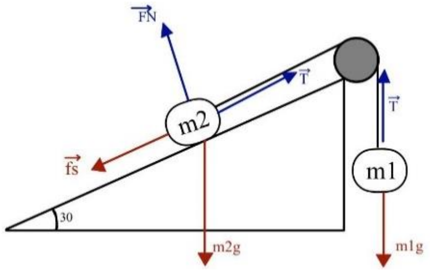
★ ***Question 7***: A box of mass is given an initial speed of 12 m/s up a 30° inclined plane. If the coefficient of kinetic friction between the box and the plane is $\mu_k = 0.15$. Find the maximum distance the box moves up the inclined plane.



☑ ***Solution:*** (d = 11.7 m)

✓ ***Example :***In the figure μ_k between $m_1$and m_2 , and the inclined plane is 0.2 . Find the speed of m_1 after fallen a distance of 1.5 m.(Assume system started from rest, $m_1 = 4kg$ and $m_2 = 2\, kg$)

✓ ***Solution:*** $W_{NC} = \Delta\, KE + \Delta\, U$



$W_{NC} = W_t + W_{fk}$

$= W_{TV} + W_{Th} + W_{fk}$

$= -Td + Td + W_{fk}$

$= 0 + W_{fk}$

$= W_{fk}$

$W_{fk} = \Delta\, KE + \Delta\, U$

$$f_k\, (1.5) \cos 180^\circ = \left[\frac{1}{2} m_1 ({v_f}^2 - v_i^2) - m_1 g(1.5)\right] + \left[\frac{1}{2} m_2 ({v_f}^2 - v_i^2) - m_2 g(1.5 \sin 30^\circ)\right]$$

$$-\mu_k\, m_2 g \cos 30^\circ = \left[\frac{1}{2} m_1 ({v_f}^2 - 0) - m_1 g(1.5)\right] + \left[\frac{1}{2} m_2 ({v_f}^2 - 0) - m_2 g(0.75)\right]$$

While m_1 and m_2 are connected by a string , they have the *same speed*

$$-0.2 * \not{2} * 9.8 * \frac{\sqrt{3}}{\not{2}} = \frac{1}{2} (m_1 + m_2)\, {v_f}^2 + (-44.1)$$

$$-3.394 = 3\, {v_f}^2 - 44.1$$

$$3\, v_f^2 = 40.70 \longrightarrow v_f = 3.68 \text{ m/s}$$

❖ Section (6.10): Power

- Power: is defined as the rate at which work is done, It could also be defined as the rate at which energy is transformed It's a scalar quantity.
- Average power: The work done divided by the time to do it.

$$\bar{P} = \frac{Work\ done}{Time\ taken}$$

- Unit of power(P) is $\frac{J}{s} = Watt\ (w)$, There is another unit which is the horse power (hp)

1 hp = 746 watt

✓ ***Example***: A 60 kg firefighter climbs a 10 m vertical rope in 10 seconds at a *constant speed*, Find his average power output

✓ ***Solution:***

$$\bar{P} = \frac{W_f}{t} = \frac{F\ d\cos\theta}{t} = \frac{f\ d}{t} \quad ((\ \theta = 0^{\circ}\ ))$$

To find F , we must use the newton's second law in vertical (y)

$\sum F_y = m\,a_y$ (constant speed → $a_y = 0$)

$\sum F_y = 0 \quad \rightarrow \quad F - mg = 0$

$\therefore\ F = mg$

$$\bar{P} = \frac{m\,g\,d}{t} = \frac{60*9.8*10}{10} = 588\ W$$

- ***Another case*** $P = \frac{W}{\Delta t} = \frac{|\vec{F}| * |\vec{d}|\ \cos\theta}{\Delta t}$

$$\boldsymbol{P = |\vec{F}|\ |\vec{v}|\cos\theta}$$

✓ ***Example***: A constant friction force f is retarding the motion elevator of mass m.

I. How much power must a motor deliver to lift the elevator at a constant speed v ?

II. What power must the motor deliver at the instant speed of the elevator is v if the motor is designed to provide the elevator with an upward acceleration a ?

✓ ***Solution:***

I. $F_{motor} = mg + f$

$\therefore\ P_{motor} = F * v = (mg + f\ )\,v$

f **increases** the power necessary to lift the elevator.

II. $F_{motor} - f - mg = ma \rightarrow F = m\,(\,a + g\,) + f$

$\therefore\ P_{motor} = F * v = [\,m\,(\,a + g\,) + f\,]\,v$

Problems

9. A box of mass 4.0 kg is accelerated from rest by a force across a floor at a rate $2m/s^2$ of for 7.0s Find the net work done on the box.

☑ **Solution:**

- ✓ The work done (w) on the box is equal to the change in kinetic energy:

$$W = \Delta KE = KE_{final} - KE_{initial}$$

- ✓ Since the box starts from rest, $KE_{initial} = 0$: $W = KE_{final} = \frac{1}{2}mv^2$
- ✓ v = a*t = 14 m/s
- ✓ w = 392J

10. A 380-kg piano slides 2.9 m down a 25° incline and is kept from accelerating by a man who is pushing back on it parallel to the incline (Fig.). Determine:

(a) the force exerted by the man

(b) the work done on the piano by the man

(c) the work done on the piano by the force of gravity

(d) the net work done on the piano. Ignore friction

☑ **Solution:**

(a) To find the force exerted by the man, we first need to determine the gravitational force acting on the piano along the incline. The gravitational force can be resolved into two components: one parallel to the incline and one perpendicular to the incline:

- ✓ Calculate the weight of the piano:

$$W = m \cdot g = 380\ kg \cdot 9.81\ m/s^2 \approx 3730\ N$$

- ✓ Calculate the component of the weight acting parallel to the incline:

$$F_{gravity,\ parallel} = W \cdot \sin(\theta) = 3730\ N \cdot \sin(25^\circ)$$

$$F_{gravity,\ parallel} \approx 1576.6N$$

- ✓ Since the piano is in equilibrium (not accelerating), the force exerted by the man F_{man} must balance the gravitational force parallel to the incline:

$$F_{man} = F_{gravity,\ parallel} \approx 1576.6N$$

(b) The work done W_{man} by the man is calculated as:

$$W_{man} = F_{man} \cdot d$$

- ✓ Since the force exerted by the man is in the direction opposite to the displacement:

$$W_{man} = -F_{man} \cdot d \approx -4572.14J$$

(c) The work done $W_{gravity}$ by gravity can be calculated as:

$$W_{gravity} = F_{gravity,\ parallel} \cdot d$$

- ✓ Since the gravitational force acts in the direction of the displacement:

$$W_{gravity} = F_{gravity,\ parallel} \cdot d \approx (1576.6\ N) \cdot (2.9\ m) \approx 4572.14\ J$$

(d) The net work done W_{net} on the piano can be calculated by summing the work done by the man and the work done by gravity:

$$W_{net} = W_{man} + W_{gravity}$$

- ✓ Substituting the values:

$$W_{net} = -4572.14\ J + 4572.14\ J = 0\ J$$

18. How much work must be done to stop a 925-kg car traveling at 95 km/h?

☑ **Solution**:

- ✓ V = 95 km/h ≈ 26.39 m/s
- ✓ $KE = \frac{1}{2}mv^2 \approx 462{,}176.36$ J
- ✓ The work W required to stop the car is equal to the negative of the initial kinetic energy, since the car is brought to rest and its final kinetic energy is zero:

$$W = -\Delta KE = -KE$$

$$W \approx -462{,}176.36 \text{ J}$$

23. A 265-kg load is lifted 18.0 m vertically with an acceleration a = 0.160g by a single cable. Determine:

(a) the tension in the cable

(b) the net work done on the load

(c) the work done by the cable on the load

(d) the work done by gravity on the load

(e) the final speed of the load

assuming it started from rest

☑ **Solution:**

(a) To find the tension in the cable, we apply Newton's second law. The forces acting on the load are the tension T pulling it upwards and the weight mg acting downwards.

- ✓ $F_{net}=ma$

  $T-mg=ma$

  $T = ma+mg=m(a+g)$

  $T = 3012.52N$

(b) The net work W_{net} is given by the change in kinetic energy of the load. Since the load starts from rest, the net work is:

- ✓ $F_{net} = T\text{-}mg = ma = 415.52N$

  $W_{net} = F_{net}\, d \cos(\theta) = 7479.36$ J

(c) The work done by the tension force in the cable is given by:

- ✓ $W_T = T\cdot d = 54225.36J$

(d) The work done by gravity is given by:

- ✓ $W_g = -mgd = -46746J$

(e) To find the final speed v, use the kinematic equation: $v^2= v_0^2 + 2ad$ Since $v_0=0$, this simplifies to: $v^2=2ad$

- ✓ The acceleration $a=0.160g=0.160\times9.8$ m/s^2 so : $a=1.568$ m/s^2
- ✓ Now, calculate v^2: $v^2=2\times1.568\times18.0=56.448$ Taking the square root: $v=\sqrt{56.448}=7.51$ m/s
- ✓ **So**, the final speed of the load is $v=7.51$ m/s.

28. A 1.60-m-tall person lifts a 1.65-kg book off the ground so it is 2.20 m above the ground. What is the potential energy of the book relative to

(a) The ground?

(b) The top of the person's head?

(c) How is the work done by the person related to the answers in parts (a) and (b)?

☑ **Solution:**

(a) The potential energy U relative to the ground is calculated using the formula: U = mgh = 35.508J

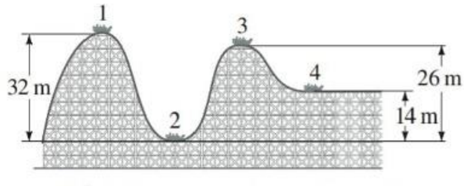
(b) To calculate the potential energy relative to the top of the person's head, we need to find the height difference between the book and the top of the person's head. The height difference is:

- ✓ $h_{difference}=h_{book}-h_{person} = 2.20 - 1.60 = 0.60$
- ✓ $U_{head} = mgh_{difference}$ = 9.702J

(c) The work done by the person in lifting the book is equal to the change in potential energy of the book, as work is the energy transferred by force over a distance. The work done is the same in both cases, but the potential energy is measured relative to different reference points.

- ✓ The work done by the person is equal to the potential energy relative to the ground (35.51 J) because the book is lifted from the ground to a height of 2.20 m.
- ✓ The potential energy relative to the top of the person's head (9.70 J) is smaller because the reference point (the top of the head) is 1.60 m above the ground. However, the work done remains the same because the energy required to lift the book through the vertical distance (0.60 m) from the head is part of the total work.

36. A roller-coaster car shown in Fig. is pulled up to point 1 where it is released from rest. Assuming no friction, calculate the speed at points 2, 3, and 4.



☑ **Solution:**

- ✓ No friction so $W_{friction} = 0$
- ✓ N is perpendicular to displacement ⟶ $W_N = 0$
- ✓ Only force along work is the weight mg which is a conservative force : $\Delta KE + \Delta PE = 0$

  $\frac{1}{2}m(v_2{}^2 - v_1{}^2) - mgh$ (where v_1 =0 , h = 32)
- ✓ $V_2 = \sqrt{627.2} \approx 25.043 m/s$
- ✓ To find v_3

  $\frac{1}{2}m(v_3{}^2 - 0) - mg(32 - 26)$ (where v_1 =0 , h_1 = 26, h_2 = 32)
- ✓ $V_3 = \sqrt{117.6} \approx 10.844 m/s$
- ✓ To find v_4

  $\frac{1}{2}m(v_4{}^2 - 0) - mg(32 - 14)$ (where v_1 =0 , h_1 = 14, h_2 = 32)
- ✓ $V_4 = \sqrt{352.8} \approx 18.78 m/s$

41. A 16.0-kg child descends a slide 2.20 m high and, starting from rest, reaches the bottom with a speed of 1.25m/s How much thermal energy due to friction was generated in this process?

☑ **Solution**:

$$\Delta k + \Delta U = W_{nc}$$
$$W_{nc} = 0.5\,(16\,)(1.25)^2 - 0 - (16)g(2.2)$$
$$-332.5$$

✓ The thermal energy generated due to friction is 332.5 J

44. A skier traveling 11m/s reaches the foot of a steady upward 19° incline and glides 15 m up along this slope before coming to rest. What was the average coefficient of friction?

☑ **Solution:**

$$\Delta k + \Delta U = W_{nc}$$
$$W_{nc} = 0.5\,m(0) - (11)^2 + mg(d\sin 19^{\circ}) = \ f_k(15)\cos 180^{\circ}$$
$$\text{When } f_k = \ \mu_k F_N \text{ when } F_N = mg\cos 19^{\circ}$$
$$\mu_k \approx 0.092$$

✓ The average coefficient of friction μ is approximately 0.092

55. How much work can a 2.0-hp motor do in 1.0 h?

☑ **Solution:**

1 hp = 746 watt ,, 1h = 3600s

$$P = \frac{W}{t}$$

2 (746) = $\frac{W}{3600}$ = 5371200 J

✓ The 2.0-hp motor can do 5.37 MJ

ARKAN ACADEMY

علم في كل مكان

Arkan academy

Arkanacademy

www.arkan-academy.com

+962 790408805